Defense Technical Information Center Compilation Part Notice

ADP015018

TITLE: One-Dimensional Nonlinear and Nonlocal Oscillations of Plasma

DISTRIBUTION: Approved for public release, distribution unlimited

This paper is part of the following report:

TITLE: International Conference on Phenomena in Ionized Gases [26th] Held in Greifswald, Germany on 15-20 July 2003. Proceedings, Volume 4

To order the complete compilation report, use: ADA421147

The component part is provided here to allow users access to individually authored sections of proceedings, annals, symposia, etc. However, the component should be considered within the context of the overall compilation report and not as a stand-alone technical report.

The following component part numbers comprise the compilation report:
ADP014936 thru ADP015049

UNCLASSIFIED

One-dimensional nonlinear and nonlocal oscillations of plasma

Paulius Miskinis
Vilnius Gediminas Technical University, Faculty of Fundamental Sciences, Department of Physics, Sauletekio Ave. 11, LT-2040, Vilnius, Lithuania

A simple one-dimensional model describing nonlinear and nonlocal oscillations of electron plasma is considered. Due regard to the nonlocality of field potential allows to smooth down the peaks of nonlinear waves, to steep up the wave front, to modify the velocity amplitude and phase of the travelling wave.

1. Introduction

In many of the models of election magnetic hydrodynamics, a more complex spatial distribution of currents and fields is usually reduced by shifting to the corresponding two- or even one-dimensional systems, utilizing the symmetry of a model under study. Nevertheless, the magnetic field is essentially three-dimensional, and such simplifications can evoke manifestations of the effective nonlocality of a two- or one-dimensional medium. A good example of such nonlocality can be electron flows in a flat plasma layer [1], the Hall effect in thin films [2], in which the nonlocality of the medium is manifested in an integral relation between the current density J and the z-component of the magnetic field B_z expressed by the Ampere law:

$$B_z = \text{v.p.}\frac{2}{c}\int_{-\infty}^{+\infty}\frac{J(\xi)}{\xi - x}d\xi \tag{1}$$

The nonlocality of vortex filaments in solitary thin film [3, 4] and layered superconductors [5], where nonlocality is manifested in the expression for the two-dimensional current function ψ related to the non-divergent current density j (e_z is the unit vector of the axis z)

$$j = \frac{c}{4\pi}\nabla\times(\psi e_z). \tag{2}$$

In the present work, on an example of a simple one-dimensional model, the problem of nonlinear and nonlocal oscillations of electron plasma is investigated. Due regard to the nonlocality of the field potential allows to smooth down the peaks of nonlinear waves, to steep up the wave front, to modify the velocity amplitude and to change the velocity phase of the travelling wave.

2. The nonlocal oscillations

The simplest model of electron plasma oscillation, which takes into account the influence of the potential nonlocality, is described by the model system of equations

$$\begin{cases} \dfrac{\partial\rho}{\partial t} + \dfrac{\partial(\rho v)}{\partial x} = 0, \\ \dfrac{\partial v}{\partial t} + v\dfrac{\partial v}{\partial x} = {}_aD_x^\alpha\varphi, \\ \dfrac{\partial^2\varphi}{dx^2} = 4\pi(\rho - 1), \end{cases} \tag{3}$$

where ρ is the density of plasma, v is the velocity, and φ is the potential. The system of units $e = m = \rho_0 = 1$ has been chosen. ${}_aD_x^\alpha\varphi$ is a fractional derivative [6, 7]:

$${}_aD_x^\alpha f(x) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dx}\int_a^x\frac{t(t)dt}{(x-t)^\alpha} \tag{4}$$

and the order of the derivative $\alpha \sim 1$. Note here that at $\alpha = 1$ we have a classic system of evolution equations described in [8, 9].

We will take into consideration only the travelling wave type solutions that depend only on the variable $\xi = x - ut$, where $u = const$ is the wave velocity. This assumption essentially simplifies the evolution equation (1) and leads to the system

$$\begin{cases} \dfrac{\partial}{\partial\xi}\rho(v-u) = 0, \\ \dfrac{\partial}{\partial\xi}\left(\dfrac{1}{2}v^2 - uv - {}_aI_x^{1-\alpha}\varphi\right) = 0, \\ \dfrac{\partial^2\varphi}{d\xi^2} = 4\pi(\rho - 1), \end{cases} \tag{5}$$

where ${}_aI_x^\alpha = {}_aD_x^{-\alpha}\varphi$ is the fractional integral of the order α [6].

From the first equation in (5) follows the relation

$$\rho(v-u) = A, \tag{6}$$

where $A = const$. It plays the role of the boundary condition. Also, from the other two equations of the system (5) it follows that

$$v = u - \sqrt{2\,{}_aI_x^\alpha\varphi} \tag{7}$$

and the Hamiltonian of the system

$$H = \frac{1}{2}v^2 - uv - {}_aI_x^{1-\varepsilon}\varphi = const \tag{8}$$

Let us introduce the new variables

$$x=\frac{1}{4}v^2,\quad \frac{1}{2}y^{3-\alpha}=-{}_aI_x^{1-\alpha}\varphi\,. \tag{9}$$

Then the Hamiltonian H is

$$H=\frac{1}{2}y^{3-\alpha}-2u\sqrt{x}+2x=-\frac{1}{2}u^2\,. \tag{10}$$

The phase trajectories in the phase plane (x, y) (see Fig.1) are closed curves surrounding the centre-focus (0, $u^2/4$):

$$y^{3-\alpha}=4u\sqrt{x}-4x+2E\,, \tag{11}$$

and crossing the axis x in the points

$$x=\frac{1}{4}(u\pm s)^2,\quad s=\sqrt{u^2+2E}\,. \tag{12}$$

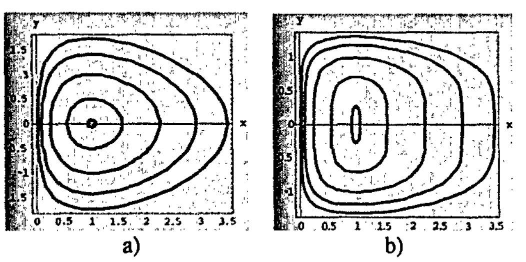

Fig. 1. Phase diagram of a system with the Hamiltonian $H=y^{3-\alpha}/2-2u\sqrt{x}+2x$: a) for α = 1 (local case) and b) α = 0.6 (nonlocal case); $u=2$ and $E=-3.995$; -3; -2; -1.

The bound states $E<0$ are separated from the unbound ones by the condition $E=0$, which leads to the region $x\in[0;u^2]$.

It is possible to show that the period of plasma oscillation for $\alpha=1$

$$T=\oint\frac{dv}{\dot{v}}=\frac{2\pi}{\omega_o} \tag{13}$$

and does not depend on the oscillation energy E. At the same time the oscillations are unharmonic.

Interestingly, after substituting (7) the evolution equations turn into a system of separate equations:

$$\begin{cases}\rho=-\dfrac{A}{\sqrt{2{}_aI_\xi^{1-\alpha}\varphi}},\\ v=u-\sqrt{2{}_aI_\xi^{1-\alpha}\varphi}\,,\\ \dfrac{d^2\varphi}{d\xi^2}=-4\pi\left(A\varphi^{-\frac{1}{2}}+1\right).\end{cases} \tag{14}$$

Nevertheless, the velocity $v(\xi)$ can be derived from the Hamiltonian H (10). If $\psi(\xi)=v(\xi)/\varepsilon$, where $\varepsilon=\left(1-|E|/(2\omega_o^2)\right)$, then

$$\psi(\xi)=-\cos\left[\pm\omega_o\xi-\varepsilon\left(1-\psi^2\right)^{\frac{1}{3-\alpha}}\right], \tag{15}$$

where $\omega_o^2=4\pi e^2n_o/m$ is the frequency of plasma oscillations, and "±" stands for the different half-periods of oscillations. The explicit dependence $v(t)$ for a fixed value of the space variable x is shown in Fig.2. For $\alpha=1$, we have only local oscillations:

$$\psi=-\cos\left[\pm\omega_0\xi-\varepsilon\left(1-\psi^2\right)^{1/2}\right],\quad \psi=\frac{1}{\varepsilon}v\,.$$

3. Conclusions

The main peculiarities of the nonlocal plasma velocity oscillations are:

- steepening up of the wave front
- changes of the velocity amplitude $v\sim u-\omega_0^{\frac{\alpha-1}{2}}\sqrt{2\varphi}$
- the smoothened peak of the nonlinear wave top
- the global phase shift $\Delta\psi\sim(\alpha-1)\pi/2$ of velocity oscillations

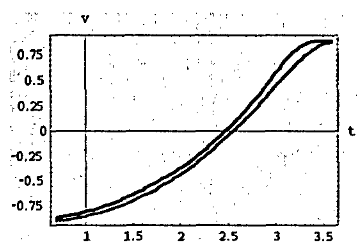

Fig. 2. Velocity of oscillations $v(t)$ in dependence on time at the frequency $\omega=1$, parameter $\varepsilon=0.9$, if nonlocality $\alpha=0.05$ and phase shift $\Delta\varphi=0.079$ (upper line) respect the local case (lower line). For simplicity, the next symmetric half-period of the periodic wave is omitted.

Note that all these peculiarities are of qualitative character. For quantitative assessments, a model of real plasma is advisable.

4. References

[1] E.B. Tatarinova, K.V. Chukbar. JETP **92**(3) (1987) 809.

[2] K.V. Chukbar. JETP **97**(4) (1990) 1362 .

[3] A.N. Artemenko, A.N. Kruglov, Phys. Lett. A **143** (1990) 485.

[4] M.V. Feigel'man, V.B. Geshkenbein, A.I. Larkin. Physica C 167 (1990) 177 .

[5] K.V. Chukbar, V.B. Yan'kov. JETP Letters. **61**(6) (1995) 487 .

[6] G.S. Samko, A.A. Kilbas, O.I. Marichev. *Fractional Integrals and Derivatives.* (Gordon and Breach; Amsterdam) 1993 .

[7] R.Hilfer. *Application of Fractional Calculus in Physics.* (World Scientific, Singapore) 2000.

[8] R.Z. Sagdeev. *Problems of Theory of Plasma.* Moscow: Atomizdat. **4** (1964) 20 (in Russian) .

[9] A.I. Achiezer, G.Ya.Liubarsky. Dokl. AN USSR. **80** (1951) 193 (in Russian) .